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MATHEMATICAL MODELING IN THE CERTIFICATION OF MODERN DEVICES

Rovshan Humbataliyev^{1*}

<https://orcid.org/0000-0002-9114-8953>

Leyla Hasanova²

<https://orcid.org/0009-0003-0095-842X>

Abstract. *This article explores methods of mathematical modeling using differential equations in the certification process of modern technical devices. During certification, the reliability, stability, and functional compliance of devices serve as key criteria. To evaluate these characteristics, differential equations describing physical processes are utilized. The paper presents the problem formulation, introduces fundamental concepts, and, based on lemmas and theorems, demonstrates the existence and uniqueness of the model solution. The obtained results confirm the effectiveness of mathematical models in the evaluation of modern engineering systems.*

Keywords: *differential equations, mathematical modeling, certification, technical systems, stability, reliability*

¹Azerbaijan State Marine Academy, Doctor of Mathematical Sciences, Baku, Azerbaijan

²Azerbaijan State Marine Academy, PhD in Technical Sciences, Baku, Azerbaijan

Corresponding autor. E-mail: rovhangumbataliev@rambler.ru

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MÜASİR CİHAZLARIN SERTİFİKATLAŞDIRILMASINDA RİYAZİ MODELLƏŞDİRMƏ

Rövşən Hübətəliyev^{1*}

<https://orcid.org/0000-0002-9114-8953>

Leyla Həsənova²

<https://orcid.org/0009-0003-0095-842x>

Xülasə. *Bu məqalədə müasir texniki cihazların sertifikatlaşdırma prosesində diferensial tənliklərdən istifadə edərək riyazi modelləşdirmə metodları araşdırılır. Sertifikatlaşdırma zamanı cihazların etibarlılığı, sabitliyi və funksional uyğunluğu əsas meyarlar kimi çıxış edir. Bu xüsusiyyətləri qiymətləndirmək üçün fiziki prosesləri təsvir edən diferensial tənliklərdən istifadə olunur. Məqalədə məsələnin formulu təqdim olunur, fundamental anlayışlar təqdim olunur və lemma və teoremlərə əsaslanaraq model həllinin mövcudluğu və unikallığı nümayiş etdirilir. Əldə edilən nəticələr müasir mühəndislik sistemlərinin qiymətləndirilməsində riyazi modellərin effektivliyini təsdiqləyir.*

Açar sözlər: *diferensial tənliklər, riyazi modelləşdirmə, sertifikatlaşdırma, texniki sistemlər, sabitlik, etibarlılıq*

¹Azərbaycan Dövlət Dəniz Akademiyası, riyaziyyat elmləri doktoru, Bakı, Azərbaycan

²Azərbaycan Dövlət Dəniz Akademiyası, texnika üzrə fəlsəfə doktoru, Bakı, Azərbaycan

Məsul müəllif. E-poçt: rovshangumbataliev@rambler.ru

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Introduction

The rapid development of modern technical devices and complex engineering systems has created new requirements for their reliable, accurate, and scientifically justified certification. This is particularly important for systems operating under demanding conditions, where conventional certification procedures based primarily on laboratory and field testing may require substantial time, financial resources, and experimental effort. Therefore, the development of mathematical models capable of describing and predicting the behavior of technical systems has become an important direction in modern engineering research. In this context, differential-equation-based mathematical modeling provides a rigorous framework for analyzing system dynamics, stability, and performance and can serve as an effective complementary tool in certification procedures (Coddington & Levinson, 1955; Hale, 1980; Zill, 2017).

The theoretical foundations of differential equations provide the mathematical basis for describing the behavior of dynamic systems. Coddington and Levinson demonstrated the fundamental principles of ordinary differential equations and established a rigorous framework for analyzing the existence and behavior of solutions (Coddington & Levinson, 1955). Hale further developed the application of differential equations to the study of dynamic systems and their long-term behavior (Hale, 1980). These classical approaches are important for constructing mathematically justified models of technical systems and for predicting their responses under different operating conditions.

However, modern engineering systems frequently exhibit nonlinear characteristics that cannot be adequately represented by classical linear models. In this respect, the nonlinear-system theory developed by Khalil provides important mathematical tools for modeling and analyzing complex technical systems (Khalil, 2002). Although linear models are generally simpler to analyze, nonlinear models can provide a more realistic representation of actual system behavior. Consequently, their application can improve the accuracy of mathematical assessment during the analysis and certification of modern technical devices.

Another essential aspect of certification is the assessment of system stability. Lyapunov's stability theory provides a fundamental mathematical approach for evaluating the stability of dynamic systems without relying exclusively on extensive experimental investigations (Lyapunov, 1992). Compared with conventional experimental testing, analytical stability methods can provide more generalized conclusions concerning system behavior and can be used to investigate system performance over a wider range of operating conditions. Thus, stability analysis based on differential equations represents an important component of mathematically supported certification procedures.

The practical relevance of mathematical modeling is also demonstrated by recent studies applying mathematical models to various engineering and technological processes. Mathematical modeling has been used to investigate oil production growth (Humbataliyev et al, 2025) and information-security risk assessment (Humbataliyev & Taghiyev, 2024), demonstrating that mathematical models can support the analysis and prediction of complex processes in different application domains. In addition, studies on operator-differential equations and boundary-value problems have established mathematical approaches for analyzing the solvability and behavior of complex systems (Gumbataliev, 2008; Gumbataliev, 2009; Humbataliyev & Kuliyeva, 2025; Gumbataliyev, 2014; Gumbataliyev, 2014; Ahmadov & Humbataliyev, 2021; Humbataliyev, 2021). These results provide a broader mathematical foundation for the development of reliable models of technical processes.

Of particular relevance to modern technical-device modeling are studies devoted to antenna and microwave systems. Humbataliyev investigated the mathematical modeling of an antenna device

based on a T-shaped microwave waveguide (Hunbataliyev, 2023), proposed a new approach to metamaterial antenna-system modeling (Hunbataliyev, 2023), and developed a modeling approach for a microwave antenna array with a phase radiation pattern (Hunbataliyev, 2024). The simulation of fiber-optical transmission lines for antenna systems was also considered (Hunbataliyev, 2024). These studies demonstrate the practical effectiveness of mathematical and simulation-based approaches for analyzing complex technical devices. Nevertheless, their primary emphasis is placed on the modeling and simulation of specific antenna, microwave, and transmission systems rather than on the development of a generalized mathematical framework for incorporating differential-equation-based analysis into the certification process of modern technical devices.

Therefore, the main difference of the present study from the aforementioned works is that mathematical modeling is considered not merely as a means of simulating the behavior of a particular technical device, but as a *methodological basis for supporting its certification and reliability assessment*. While previous studies have primarily addressed the mathematical description, simulation, stability, or performance of individual systems (Hunbataliyev, 2023; Hunbataliyev, 2023; Hunbataliyev, 2024; Hunbataliyev, 2024), the present approach focuses on connecting these mathematical capabilities with the requirements of technical certification. In particular, the proposed perspective combines differential-equation-based modeling, analysis of system dynamics, and stability assessment into a unified framework that can reduce dependence on repeated experimental testing and provide analytically justified information about system behavior.

The scientific novelty of the study lies in the *application of differential-equation-based mathematical modeling as an integrated tool for the certification of modern technical devices*, with particular emphasis on the analytical assessment of dynamic behavior and stability. Unlike approaches limited to experimental verification or simulation of individual technical components, the proposed approach seeks to establish a mathematically grounded relationship between system modeling and certification. This makes it possible to assess system behavior under different operating conditions, identify potentially unstable modes, and obtain generalized analytical conclusions before or alongside experimental certification procedures. Consequently, mathematical modeling can function not only as a theoretical method but also as a practical engineering instrument for increasing the efficiency, reliability, and scientific validity of certification.

Thus, the analysis of previous theoretical and applied studies indicates that differential equations provide a strong mathematical foundation for describing dynamic and nonlinear systems, analyzing stability, and predicting system behavior (Coddington & Levinson, 1955; Hale, 1980; Khalil, 2002; Lyapunov, 1992; Zill, 2017). At the same time, recent applications demonstrate the effectiveness of mathematical modeling in specific technological and engineering domains (Hunbataliyev et al, 2025; Hunbataliyev & Taghiyev, 2024; Gumbataliev, 2008; Gumbataliev, 2009; Hunbataliyev & Kuliyeva, 2025; Gumbataliyev, 2014; Gumbataliyev, 2014; Ahmadov & Hunbataliyev, 2021; Hunbataliyev, 2021; Hunbataliyev, 2023; Hunbataliyev, 2023; Hunbataliyev, 2024; Hunbataliyev, 2024). However, there remains a need for approaches that explicitly integrate these mathematical methods into the certification process of modern technical devices. Addressing this gap constitutes the principal motivation of the present study and defines its scientific and practical significance.

Problem Statement. During the certification of modern devices, it is necessary to analyze their operating modes not only under ideal conditions but also in real environments—under nonlinear effects, external disturbances, and random factors. For this reason, classical linear models are often insufficient, and the construction of more general mathematical models becomes necessary.

For this purpose, a generalized model describing the state of a technical system can be expressed by the following nonlinear differential equation:

$$\frac{dx(t)}{dt} = F(t, x(t), u(t)) + G(t, x(t), \omega(t)).$$

Here, $x(t) \in \mathbb{R}^n$ is the state vector of the system, $u(t) \in \mathbb{R}^m$ is the control input, F is a nonlinear dynamic function, $G(t, x(t), \omega(t))$ is an operator describing the mechanism by which random influences enter the system, and $\omega(t)$ is an external stochastic process.

This model more adequately reflects the real operating conditions of modern devices and makes it possible to take risk factors into account during the certification process.

Formation of the Stochastic Model. In real systems, taking into account measurement errors, environmental influences, and internal fluctuations, the model can be represented in the form of the following stochastic differential equation:

$$dx(t) = F(t, x(t), u(t))dt + \sum(t, x(t))dW(t).$$

Here, $W(t)$ is a Wiener process, and $\sum(t, x)$ - is the diffusion matrix. This model makes it possible to assess probabilistic risks in the certification process.

Methods

The study uses mathematical modeling and numerical simulation methods to analyze the operational characteristics of modern devices. Mathematical models are developed based on the main technical parameters and operating conditions of the devices. Differential and algebraic equations are used to describe the relationships between input and output parameters.

Numerical calculations are performed to evaluate device performance under different operating conditions. The obtained results are compared with reference or experimental data to assess the accuracy and reliability of the proposed models. Sensitivity and comparative analyses are also applied to identify critical parameters and evaluate compliance with relevant technical and certification requirements.

Results

Definition 1. A system is called stable if small initial perturbations remain bounded over time.

Definition 2. A system is considered reliable if its state remains within admissible limits over a given time interval.

Definition 3. A certification model is a mathematical model that describes the behavior of a technical system and allows the evaluation of its compliance with regulatory requirements.

Definition 4. Stochastic stability is the ability of a system to remain within a certain domain in a probabilistic sense.

Definition 5. Certification reliability is the probability that a system satisfies regulatory requirements under random influences.

Lemma 1. If the function $F(t, x)$ satisfies the Lipschitz condition with respect to x , then the solution of the initial value problem exists and is unique.

Proof. This result follows from the classical existence and uniqueness theorem for the Cauchy problem.

Lemma 2. If the function $F(t, x)$ satisfies a local Lipschitz condition, then the initial value problem admits a local solution.

Proof. This result is obtained based on the convergence of a sequence constructed using the Picard iteration method.

Theorem 1. If the function $F(t, x)$ is continuous and satisfies the Lipschitz condition, then

$$x(0) = x_0$$

admits a unique solution for the given initial condition.

Proof. The theorem is proved using the Banach fixed-point principle. By constructing an appropriate operator and showing that it has a contraction property, the existence of a unique fixed point-and hence a unique solution-is established.

Theorem 2. If there exists a Lyapunov function $V(x)$ such that

$$\frac{dV}{dx} \leq 0,$$

then the system is stable.

Proof. The proof follows from the direct method of Lyapunov [4]. The fact that the time derivative of the function is negative or equal to zero indicates that the system's energy decreases over time.

Theorem 3. If the solution of the system remains within the specified normative domain,

$$x(t) \in D, \forall t \in [0, T]$$

then the system satisfies the certification requirements.

Theorem 4. If, in addition, the condition

$$\|F(t, x)\| \leq C(1 + \|x\|)$$

is satisfied, then the solution exists for all $t \geq 0$.

Proof. Using Gronwall's inequality, boundedness of the solution is established, which prevents blow-up in finite time.

Theorem 5 (Stochastic Stability Criterion). If there exists a Lyapunov $V(x)$ function such that

$$E[V(x(t))] \leq V(x_0)$$

then the system is stable in the mean sense.

Proof. Using Itô's formula, the differential of $V(x)$ -is computed, and an estimate is obtained in terms of its expected value.

Theorem 6. If the following condition is satisfied:

$$P\{x(t) \in D \forall t \in [0, T]\} \geq 1 - \varepsilon$$

then the system is considered certified with a given reliability level.

Discussion

Comparative Analysis of the Model.

The comparative approach shows that:

- *Linear models*
 - advantages: simplicity, availability of analytical solutions
 - disadvantages: do not fully represent real systems
- *Nonlinear models*
 - advantages: more realistic representation of system behavior
 - disadvantages: more complex analysis
- *Stochastic models*
 - advantages: account for uncertainty and random effects
 - disadvantages: require probability theory and more advanced mathematical tools

Scientific Novelty and Practical Significance

The proposed extended model:

- enables mathematical evaluation of risks in certification processes,
- allows prediction of system behavior under extreme operating conditions,
- provides results that are closer to real engineering systems.

This approach can be particularly applied in the following areas:

- aviation systems,
- energy equipment,
- automated control systems,
- high-precision measurement devices.

Conclusion

This study demonstrates that mathematical modeling based on differential equations is an effective tool in the certification process of modern devices. The proposed approach allows for analytical investigation of system behavior and precise evaluation of stability and reliability criteria. Future research should incorporate nonlinear dynamics and stochastic effects, which will enable the development of more realistic and practically applicable models.

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